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GRAVITATIONAL EXPERIMENTS WITH A COLLISIONLESS TWO-DIMENSIONAL COMPUTER MODEL

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SUMMARY

A two-dimensional model is used to perform computer experiments for a collisionless self-gravitating system. The gravitational field is obtained by solving the Poisson equation and the system is advanced stepwise in time. Computer simulations have been performed for systems with an initially uniform distribution over a circular region in x,y space, zero thermal velocity, and various values of initial solid-body rotation. Up to 4000 stars were used in the calculations.

INTRODUCTION

Actual stellar systems, such as galaxies, contain about 10^{11} stars with a sufficiently large average separation so that binary encounters between stars can be neglected. Stellar systems can therefore be described by the collisionless Boltzmann equation. In order to simulate the evolution of such stellar systems on a computer, the motion of at least several thousand masses or stars should be followed. Hohl and Feix (refs. 1 and 2) and Lecar and Cohen (ref. 3) used one-dimensional sheet models to study the evolution of self-gravitating systems. A similar model where the motion of a large number of concentric spherical mass shells is followed has been used by Hénon (refs. 4 and 5). Recently, Hockney* (ref. 6) and Hohl (refs. 7 and 8) introduced a two-dimensional model where the stars are represented by infinitely long mass rods. In the present report the results of some simple experiments with the two-dimensional model are given. The calculations show that filamentary and other irregular structures similar to those of some actual stellar systems can be obtained by purely gravitational effects.

SYMBOLS

a_k variable defined by equation (3)

D impact parameter

*The results presented in the present report and in reference 6 are similar and they were obtained independently by the present authors and by Hockney. The methods used in the numerical calculation are different. The present method is more general and much simpler to use than the Fourier method used by Hockney.

G	gravitational constant
$i = \sqrt{-1}$	
$\vec{K}$	gravitational field, $-\nabla\varphi$
k	summation index
m, M	mass per unit length
n	star density
N	number of stars in system
P	potential energy
r	magnitude of radius vector
$\vec{R}$	radius vectors defining position
Re	real part
t	time
T	kinetic energy
U	total energy
V	velocity
$V_{\perp}$	transverse velocity
x, y	positions along X- and Y-axes
z	complex position, $x + iy$
γ	overrelaxation parameter
$\vec{\xi}$	variable defined by equation (15)
ρ	mass density
τ_c	relaxation time
τ_g	equilibrium rotation period
φ	gravitational potential

ω	frequency of rotation
ω_g	equilibrium frequency of rotation

Subscripts:

j,k,n,m	summation indexes
max	maximum
min	minimum
r	radial
x,y	x- and y-component
θ	azimuthal

Arrows over symbols denote vectors. < > denotes expectation values.

DESCRIPTION OF MODEL

The model consists of a large number of rods of equal mass per unit length and the rods are of infinite extent in the z-direction. These rods move in the x,y plane under the action of their mutual gravitational attraction. The system of mass rods is advanced in time in the following manner. First, the distribution of mass $\rho(x,y)$ is used to obtain the gravitational potential $\varphi(x,y)$ by numerically solving the Poisson equation. Second, the gravitational field at the position of the particles is computed from the potential $\varphi(x,y)$. Third, Newton's laws are used to advance the motion of all the mass rods for a small time step δt. These three steps represent one cycle and they are repeated until the desired evolution of the system is achieved.

The crucial point in the computations is the solution of the Poisson equation. It is desirable that the time required for this process be only a small fraction of the cycle time. If the system is advanced for a small time step δt, the mass distribution $\rho(x,y)$ will not change very much. The change in the gravitational potential will then also be very small. Thus, the solution of the finite difference form of the Poisson equation by an iteration method which uses the potential from the previous cycle as an initial guess will converge very rapidly. It was found that 5 to 7 iterations per cycle gave satisfactory results.

To solve the Poisson equation, the boundary conditions around the rectangular mesh used in solving the finite difference form of the Poisson equation are required. The potential at an arbitrary boundary point $z = x + iy$ can be written as

$$\varphi(x,y) = 2Gm \sum_{n=1}^{N} \log_e |z - z_n| \tag{1a}$$

where $z_n = x_n + iy_n$ is the position of a rod of mass m, G is the gravitational constant, and N is the number of particles in the system.

Equation (1a) can be written as

$$\varphi(x,y) = 2GmN \log_e |z| + 2Gm \operatorname{Re}\left[\sum_{n=1}^{N} \log_e \left(1 - \frac{z_n}{z}\right)\right] \tag{1b}$$

Since $\left|\frac{z_n}{z}\right| < 1$, the potential of the boundary points can be approximated by

$$\varphi(x,y) = 2GmN \log_e |z| - 2Gm \operatorname{Re}\left[\sum_{k=1}^{15} \frac{a_k}{kz^k}\right] \tag{2}$$

where

$$a_k = \sum_{n=1}^{N} (z_n)^k \tag{3}$$

and the series expansion for $\log_e \left(1 - \frac{z_n}{z}\right)$ has been truncated after 15 terms. The accuracy of equation (2) has been checked by direct summation of the $\log_e |z - z_n|$ potential and was found to agree to better than 0.1 percent. Equation (2) for the potential on the boundaries has also been used by Hockney (ref. 6). Equation (1a) can also be used to obtain the potential of an arbitrary point. However, the time required for such a method is too large. Therefore, the potential is obtained by solving the Poisson equation.

The Poisson equation

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 4\pi G\rho(x,y) \tag{4}$$

is solved by using the standard five-point difference equation (ref. 9)

$$\varphi_{n+1,m} + \varphi_{n,m+1} + \varphi_{n-1,m} + \varphi_{n,m-1} - 4\varphi_{n,m} = 4\pi G\rho_{n,m} \tag{5}$$

where $\Delta x = \Delta y = 1$. The mass per unit length of all the rods in the cell n,m is represented by $\rho_{n,m}$. This set of simultaneous equations is solved by an iteration method on the Control Data 6600 computer system in the form

$$\varphi^{r+1}_{n,m} = \varphi^{r}_{n,m} + \gamma\left(\varphi^{r+1}_{n-1,m} + \varphi^{r}_{n+1,m} + \varphi^{r+1}_{n,m-1} + \varphi^{r}_{n,m+1} - 4\varphi^{r}_{n,m} - 4\pi G\rho_{n,m}\right) \tag{6}$$

To save computer storage and increase the convergence rate, the new values of φ^{r+1} which have already been determined are used in the right-hand side of equation (6). The superscript r refers to the rth iteration and the parameter γ is adjusted to give the maximum rate of convergence.

The differential equations of motion for the stars are

$$\frac{d^2\vec{R}_j}{dt^2} = \vec{K}\left(R_j,t\right) \tag{7}$$

$$\frac{d\vec{R}_j}{dt} = \vec{V}_j \tag{8}$$

where $\vec{R}$ is the position of a star and $\vec{K}$ is the gravitational field. The evolution in time of the particle trajectories is given by the finite-difference approximation

$$\vec{R}_j(t + \delta t) = \vec{R}_j(t) + \vec{V}_j(t)\delta t + \frac{\delta t^2}{2}\vec{K}\left(\vec{R}_j,t\right) \tag{9}$$

$$\vec{V}_j(t + \delta t) = \vec{V}_j(t) + \vec{K}\left(\vec{R}_j,t\right)\delta t + \frac{\delta t^2}{2}\frac{d\vec{K}}{dt} \tag{10}$$

where $\frac{d\vec{K}}{dt}$ was approximated by the backward difference

$$\frac{d\vec{K}}{dt} = \frac{1}{\delta t}\left[\vec{K}\left(\vec{R}_j,t\right) - \vec{K}\left(\vec{R}_j,t - \delta t\right)\right] \tag{11}$$

Equations which were found to be nearly as accurate as equations (9) and (10) but which require fewer computer operations are (from ref. 10)

$$\vec{V}_j(t + \delta t) = \vec{V}_j(t) + \delta t \vec{K}\left(\vec{R}_j, t\right) \tag{12}$$

$$\vec{R}_j(t + \delta t) = \vec{R}_j(t) + \delta t \vec{V}_j(t + \delta t) \tag{13}$$

The term $\frac{d\vec{K}}{dt}$ in equation (10) was introduced to remove a computational instability. Similarly, the use of the new velocity in equation (13) removes an unconditional computational instability.

Equations (12) and (13) can be expressed in a simpler form by introducing the quantity

$$\vec{\xi}_j(t) = \vec{R}_j(t) - \vec{R}_j(t - \delta t) \tag{14}$$

The equations can then be written in the simplified form

$$\vec{\xi}_j(t + \delta t) = \vec{\xi}_j(t) + \vec{K}\left(R_j\right) \delta t^2 \tag{15}$$

and

$$\vec{R}_j(t + \delta t) = \vec{R}_j(t) + \vec{\xi}_j(t + \delta t) \tag{16}$$

Equations (15) and (16) were also used by Hockney (ref. 6) and they require the least number of computer operations. However, if it is desired to display the evolution of the system in velocity space or to compute the kinetic energy of the system, the equation

$$\vec{V}(t) = \left[\vec{\xi}(t) + \vec{\xi}(t + \delta t)\right] \frac{1}{2\delta t} \tag{17}$$

must be evaluated. It may then be more desirable to use equations (12) and (13). All three sets of equations were used and all gave similar results.

The kinetic energy of the system is

$$T = \frac{1}{2} m \sum_{j=1}^{N} V_j^2 \tag{18}$$

and the potential energy is

$$P = \frac{1}{2} m \sum_{j=1}^{N} \varphi\left(\vec{R}_j\right) \tag{19}$$

where the summation goes to N, the number of stars in the system. The total energy of the system U remains constant and is

$$U = T + P \tag{20}$$

For the calculations presented here, the Poisson equation was solved on a 51 by 51 mesh. During the evolution of the system, the potential at several mesh points was computed by summing directly the contribution to the potential from each mass rod. These potentials agreed with those obtained from a solution of the Poisson equation to within 0.1 percent.

Another check on the method of solution was made by comparing the evolution of two systems with identical initial conditions where the mesh size used in the solution of the Poisson equation was 51 by 51 for one system and 101 by 101 for the other. The resulting evolution of the two systems was nearly identical. The cycle time for a 2000-particle system with a 51 by 51 mesh and with 7 iterations per cycle is 1 second on the Control Data 6600 computer system at the Langley Research Center. This time includes such operations as the checking of the potential, the calculation of energy and angular momentum, and writing the positions and velocities of all stars on tape.

The relaxation time (ref. 11) for the two-dimensional model is of interest to determine the number of rotations for which the system can be treated as collisionless. Consider an encounter between a rod of mass m per unit length and velocity V with a stationary rod of mass M per unit length. If D is the distance of closest approach (impact parameter), the transverse force acting on the moving rod during a time $2D/V$ can be approximated as $2GmM/D$. (See ref. 12.) The moving star has therefore acquired a transverse velocity

$$\delta V_{\perp} \approx \frac{4GM}{V} \tag{21}$$

For a star interacting with a system containing many stars, the effect of many individual encounters must be summed. The number of encounters in a time t with impact parameter between D and $D + dD$ is $tVn\,dD$ where n is the density of stars in the system. The expectation value of ${V_{\perp}}^2$ is then given by

$$\left\langle V_{\perp}^{2}\right\rangle = \int_{D_{min}}^{D_{max}} \frac{16tnG^2M^2}{V} dD$$

$$= 16tnG^2M^2 \frac{D_{max} - D_{min}}{V}$$

$$= 16tnG^2M^2 \frac{D_{max}}{V} \tag{22}$$

since, in general, $D_{max} >> D_{min}$ where $D_{max} = \Delta x$. The relaxation time τ_c is defined as the time required for $\left\langle V_{\perp}^{2}\right\rangle$ to be of the same order of magnitude as V^2. Thus, for $\left\langle V_{\perp}^{2}\right\rangle = V^2$, $t = \tau_c$ and

$$\tau_c \approx \frac{V^3}{8G^2m^2nD_{max}^2} \tag{23}$$

where $M = m$ and D_{max} is taken to be equal to the dimension of the system, which for systems near equilibrium is the Jeans or Debye length (ref. 2).

The gravitational field inside a system with a uniform circular distribution in coordinate space is

$$K = 2\pi G\rho R$$

Balancing the gravitational force toward the center of the system by the centrifugal force $\omega^2 R$ requires a frequency

$$\omega = \sqrt{2\pi G\rho}$$

The rotational period for such a system then becomes

$$\tau_g = \frac{2\pi}{\sqrt{2\pi G\rho}}$$

For the systems investigated, the ratio of the relaxation time τ_c to the rotation period τ_g is

$$\frac{\tau_c}{\tau_g} \approx 3.5$$

Therefore, only for a time less than about 4 rotations can the system be considered as collisionless. To keep the effects of collisions negligible for the first 10 rotations requires a system containing about 10 000 stars.

For the 2000-particle systems investigated, the total energy was conserved to within 2 percent. The error in the energy for systems containing 4000 particles was less than 0.5 percent.

RESULTS AND DISCUSSION

The computer experiments were performed for systems which have an initially uniform circular distribution in x,y space and zero thermal velocity. The evolution of such systems is then studied for various values of initial solid-body rotation. The initial conditions are obtained by using a random-number generator which gives a nearly uniform distribution over a circular region of the x,y plane. The normalizations $4\pi G = 1$ and $m = 1$ were used for all the calculations.

First, the results for the case where the system is in equilibrium are presented. For this case the initial frequency of rotation ω equals ω_g, the frequency required such that the centrifugal force balances the gravitational attraction towards the center of the system. Thus $\omega = \omega_g$, where

$$\omega_g{}^2 = 2\pi G\rho$$

Figure 1 shows the evolution of a 2000-particle system in x,y coordinate space. The time t has been normalized to the period of rotation $\tau_g = \frac{2\pi}{\omega_g}$. The time step used in the calculations is $\delta t = \frac{\tau_g}{200}$. Lamb (ref. 13) has given the result that an infinitely long circular cylinder of uniform density may rotate in relative equilibrium about its longitudinal axis with an angular velocity given by $\omega^2 = \pi G\rho$. One would therefore expect the initial distribution shown in figure 1 to be unstable. Thus, at $t = 0.5\tau_g$, there appears a fourth harmonic perturbation around the periphery of the system. Later the perturbation goes to an egg shape and it finally disappears and leaves the system in a steady state. At $t = 0.75\tau_g$ and $t = 1.00\tau_g$, there is an indication of the appearance of spiral arms; however, it appears that collisional effects suppress this tendency. The evolution of the corresponding velocity distribution is shown in figure 2. There is almost no change in the velocity distribution as the system evolves in time. Figure 3 shows the velocity distribution of the system after the initial solid-body rotation has been subtracted. The radial velocity V_r is plotted against $V_\theta - r\omega_g$ where V_θ is the azimuthal velocity and r is the radius from the center of the system to a star. Initially, the thermal (or random) velocity of all mass rods is zero. Because of the small perturbation caused by the random initial position of the stars, a small thermal velocity builds up and the system expands in $V_r - (V_\theta - r\omega_g)$ space. After a time $t = 2.0\tau_g$, there is little further change in the velocity distribution. The increase in the thermal velocity causes the system to stabilize.

These calculations with $\omega = \omega_g$ were repeated for a 4000-particle system. The results are shown in figure 4. It can be seen that the deviation from a circular distribution is even smaller now than for the 2000-particle system.

The results for the case of zero initial rotation, $\omega = 0$, are presented next. Figure 5 shows the evolution of the system in coordinate space. After an initial implosion, the system expands and presents some highly filamentary structure. At $t = 0.53\tau_g$, several clusters of stars have condensed. However, at a later time only a large central cluster remains and the system takes on an appearance reminiscent of an elliptical galaxy. The corresponding evolution in velocity space is shown in figure 6. Initially, all stars have zero velocity. As the system collapses under the gravitational forces, the velocity of the stars increases. The system then expands rapidly in velocity space and shows filamentary structure similar to that appearing in coordinate space. A motion picture was prepared which shows the evolution of the system. One of the striking features that can be observed in the motion picture is the appearance of streamers in velocity space. Chains of stars move out of the main body of the system and then curve back again. After several pulsations, the pressure due to the increased temperature builds up and the system approaches an equilibrium state.

For a system with an initial rotation equal to half that required to balance gravitation, the system again contracts initially and then expands. The results are shown in figure 7(a). At time $t = 0.53\tau_g$, the system has developed an irregular structure which, however, disappears again and after only a few rotations the system approaches an equilibrium configuration similar to that of an elliptical galaxy. Figure 7(b) shows the corresponding evolution in velocity space.

In figure 8(a) the results are shown for a system with an initial rotation equal to $1.3\omega_g$. The general behavior is very similar to that of the previous case with $\omega = 0.5\omega_g$. At the time $t = 0.75\tau_g$ the system shows a structure similar to that of the Crab Nebula. The evolution in velocity space for this system is shown in figure 8(b).

The time development of the total kinetic energy of the 2000-particle systems investigated is shown in figure 9. Only small oscillations occur for the balanced case. For the other cases the initially large oscillations in the kinetic energy are quickly damped as the systems approach an equilibrium state. The rapid damping of the oscillations in the kinetic energy is another indication that collisional effects are important even after only 3 to 4 rotations.

CONCLUDING REMARKS

The two-dimensional model used to study the evolution of self-gravitating systems indicates that a variety of filamentary and other structures can be produced by purely gravitational effects. Some of the systems investigated showed a structure similar to that of the Crab Nebula.

The evolution of the balanced cylinder showed that there was a tendency for spiral structure to appear and also a tendency for the cylinder to become unstable. It appears that both of these effects are quickly suppressed by collisional effects. Thus, systems with more stars should be used to increase the ratio of the relaxation time to the rotational period. For example, to keep the system collisionless for 10 rotations requires a system containing 10 000 mass rods.

The method used for the calculations in this report is general and requires no modification either to increase the number of stars in the system or to increase the number of mesh points used in the numerical solution of the Poisson equation.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., April 16, 1968,
129-02-01-01-23.

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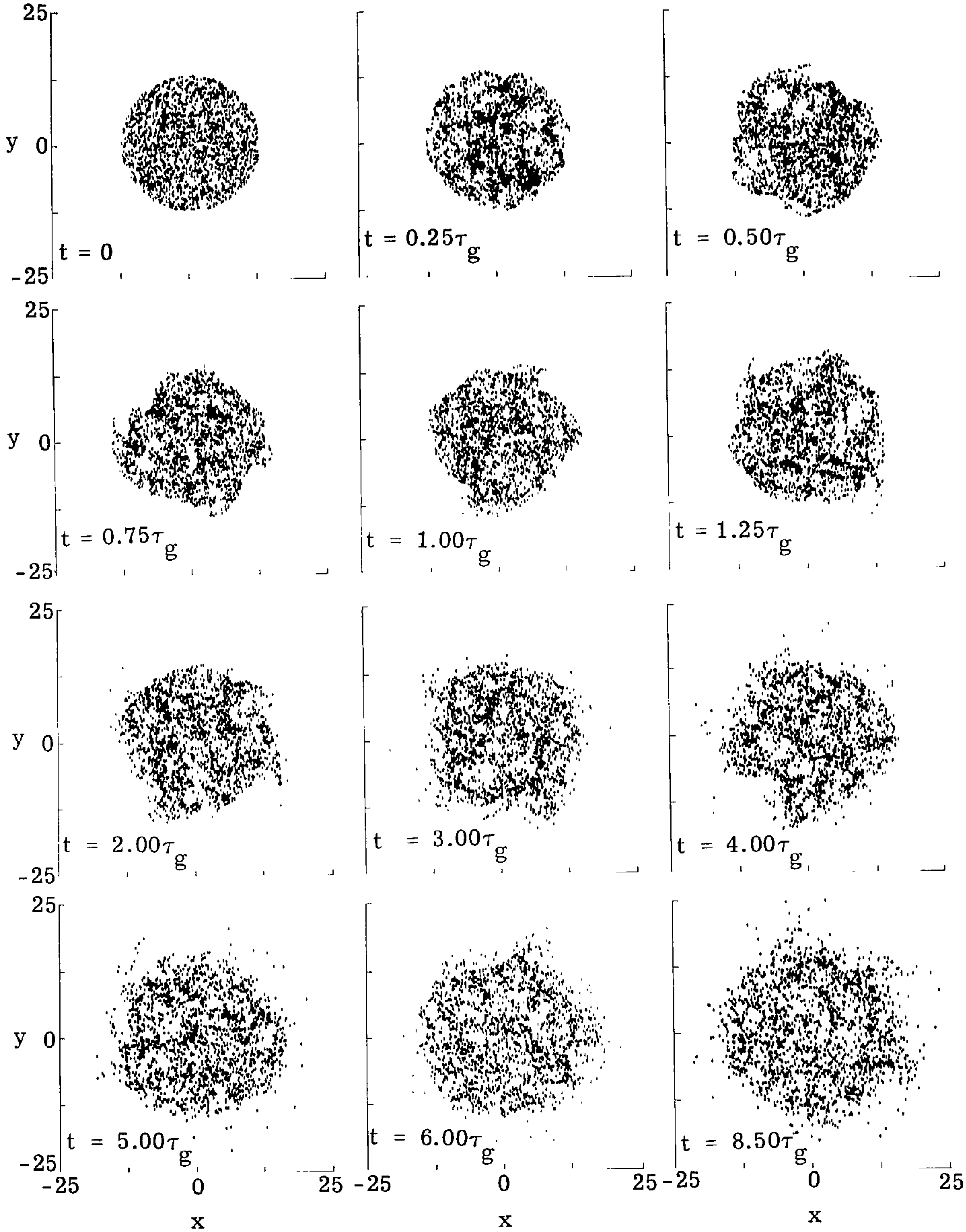


Figure 1.- Evolution in coordinate space of a cylindrical stellar system with $\omega = \omega_g$.

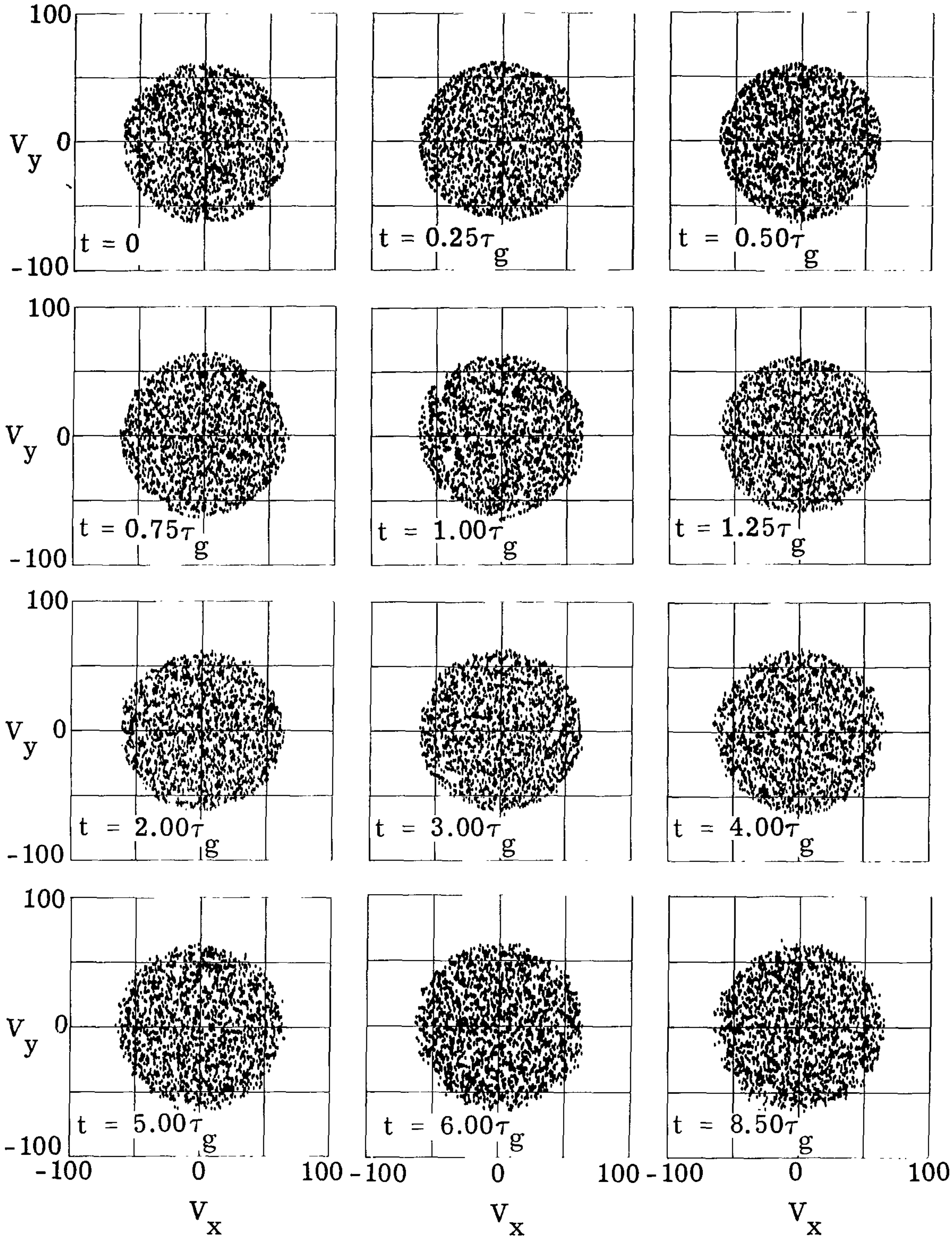


Figure 2.- Evolution in velocity space for a 2000-star system with $\omega = \omega_g$.

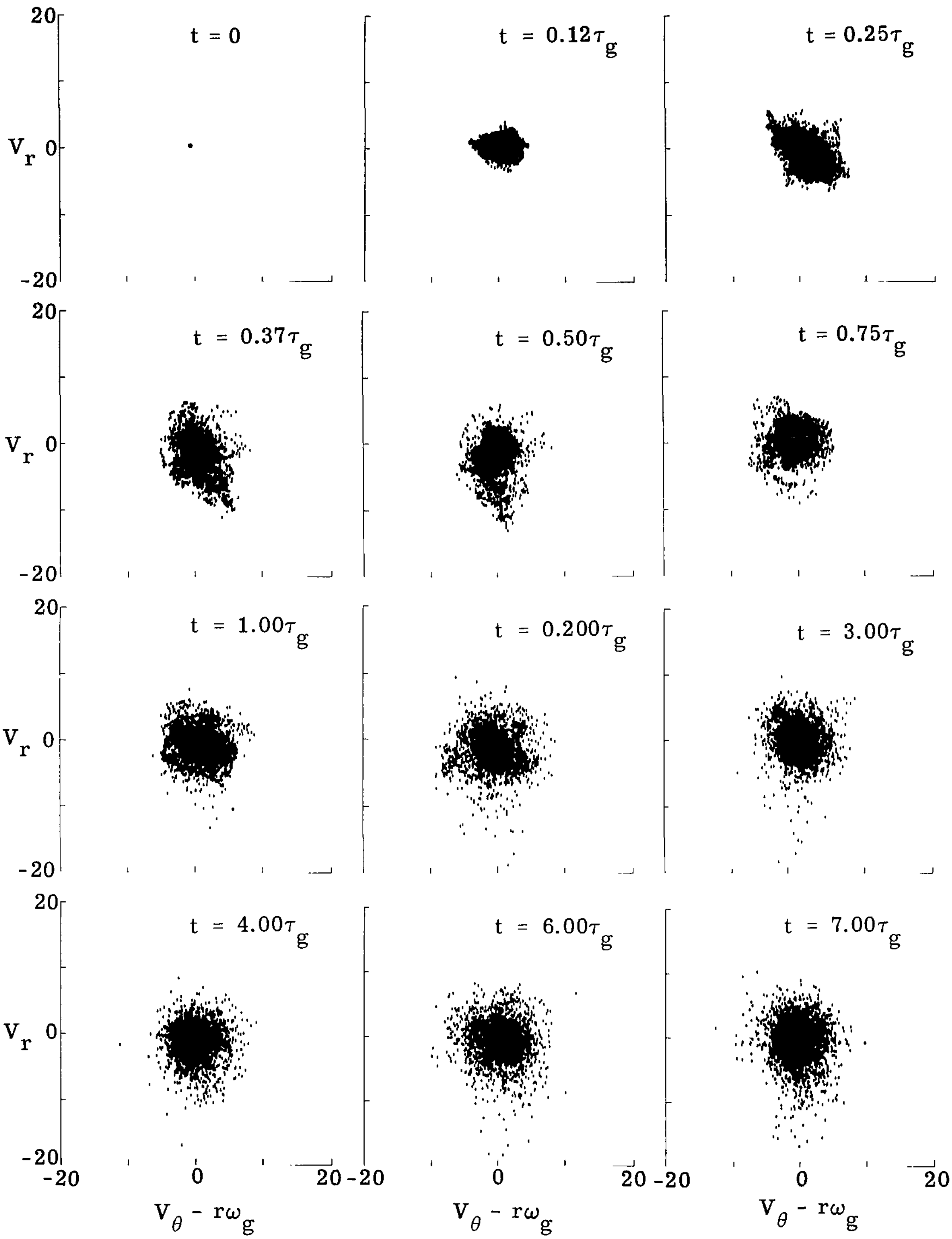
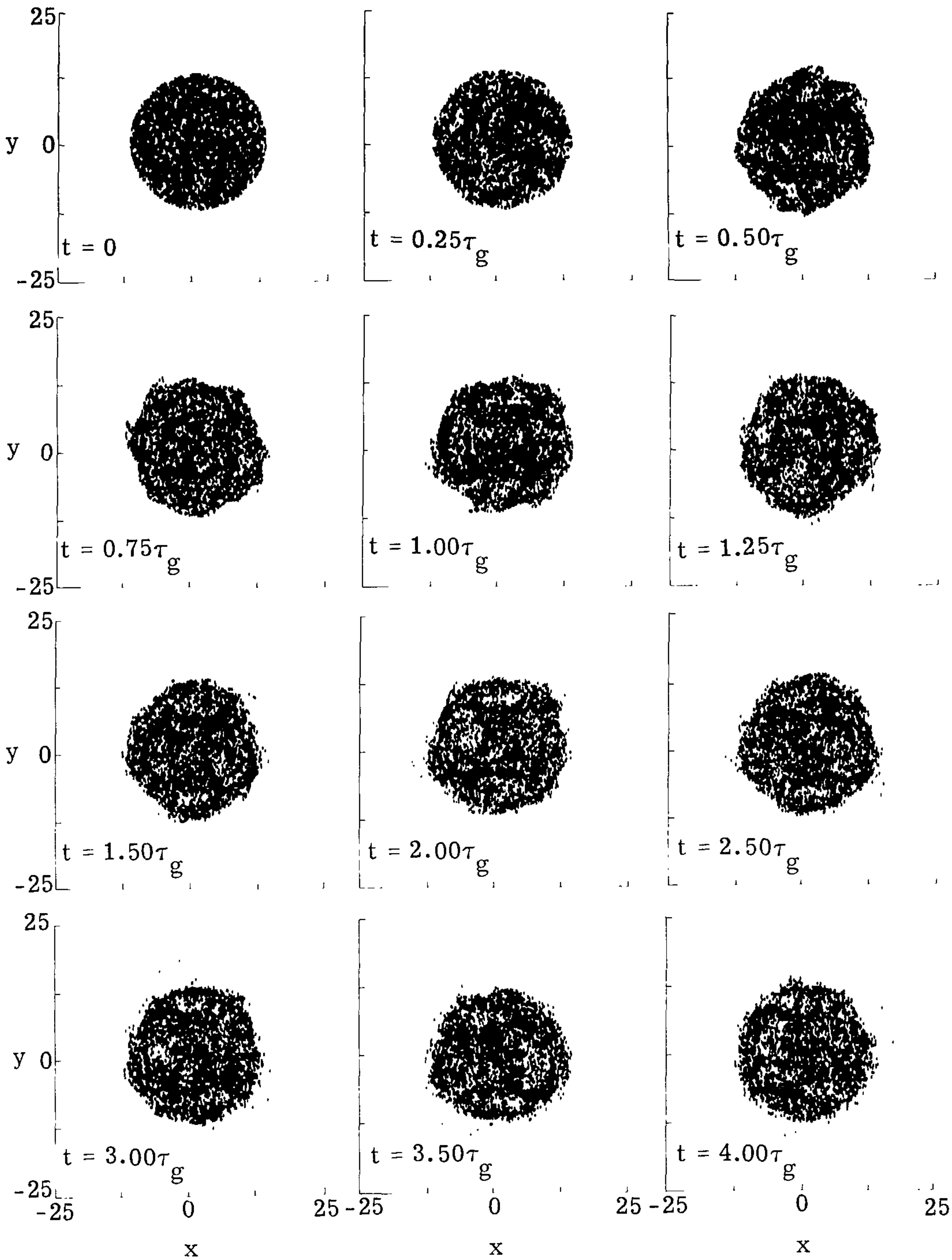
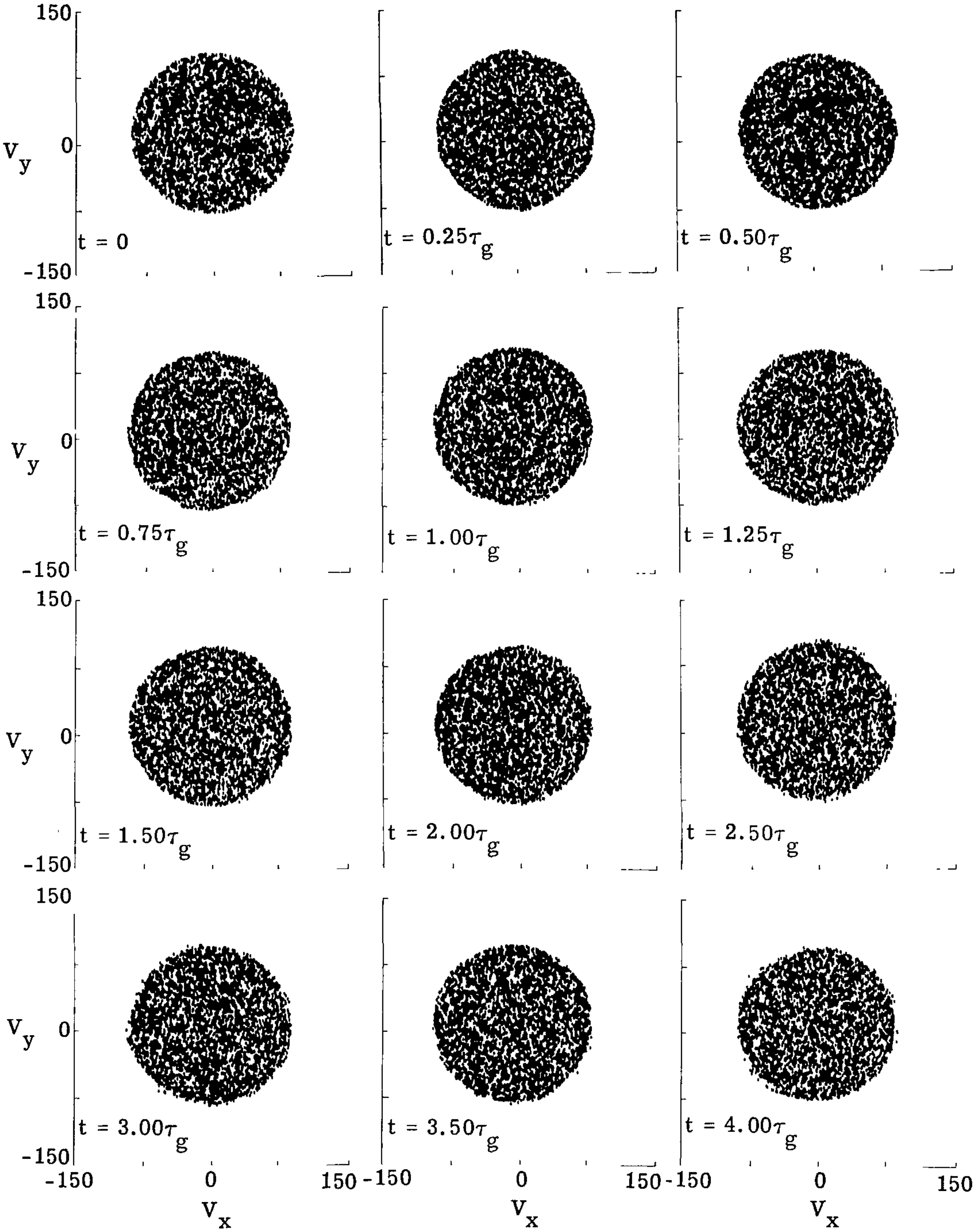


Figure 3.- Buildup of the thermal velocity for a cylindrical stellar system with $\omega = \omega_g$.



(a) In coordinate space.

Figure 4.- Evolution for a 4000-star system with $\omega = \omega_g$.



(b) In velocity space.

Figure 4.- Concluded.

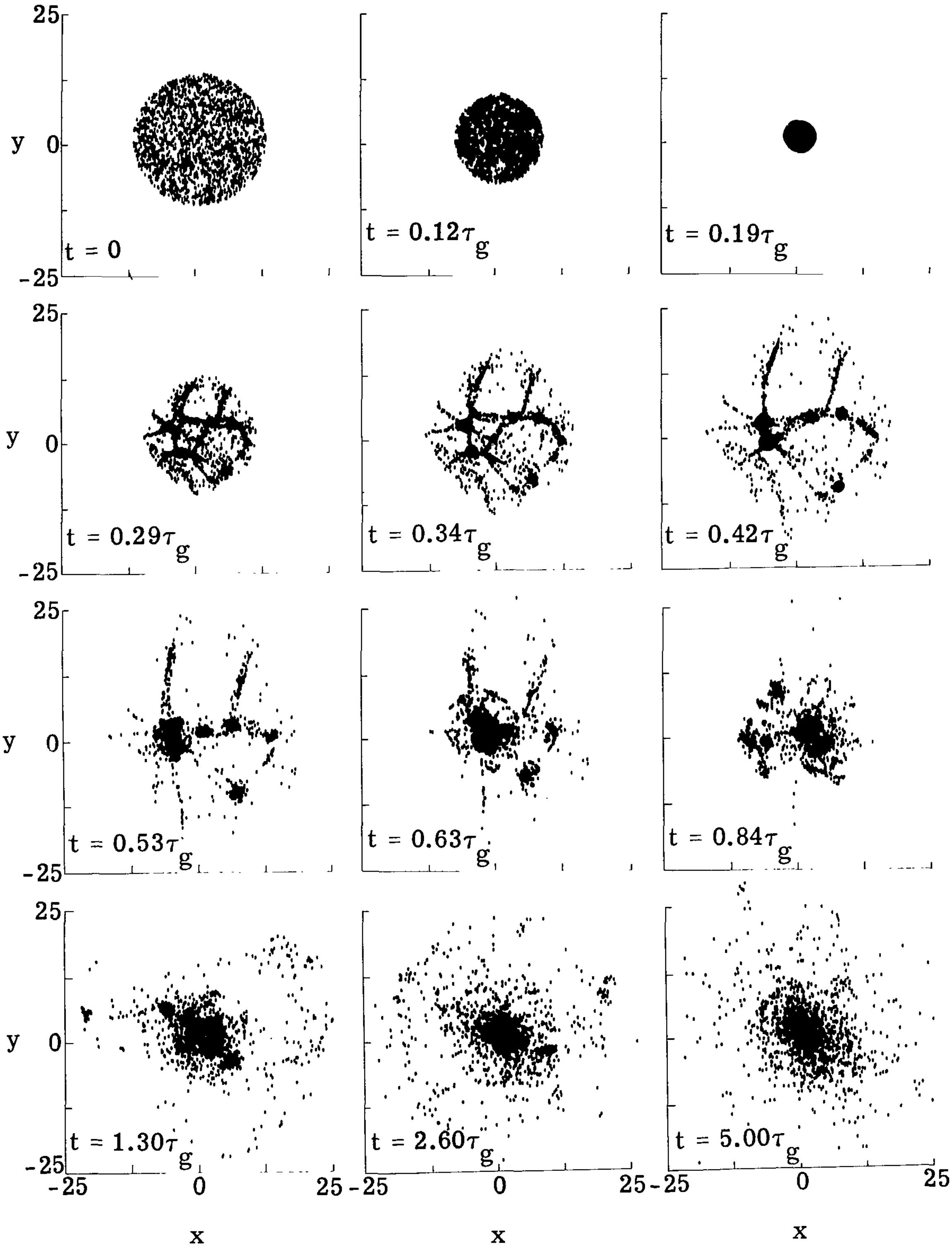


Figure 5.- Evolution in coordinate space of a cylindrical stellar system with $\omega = 0$.

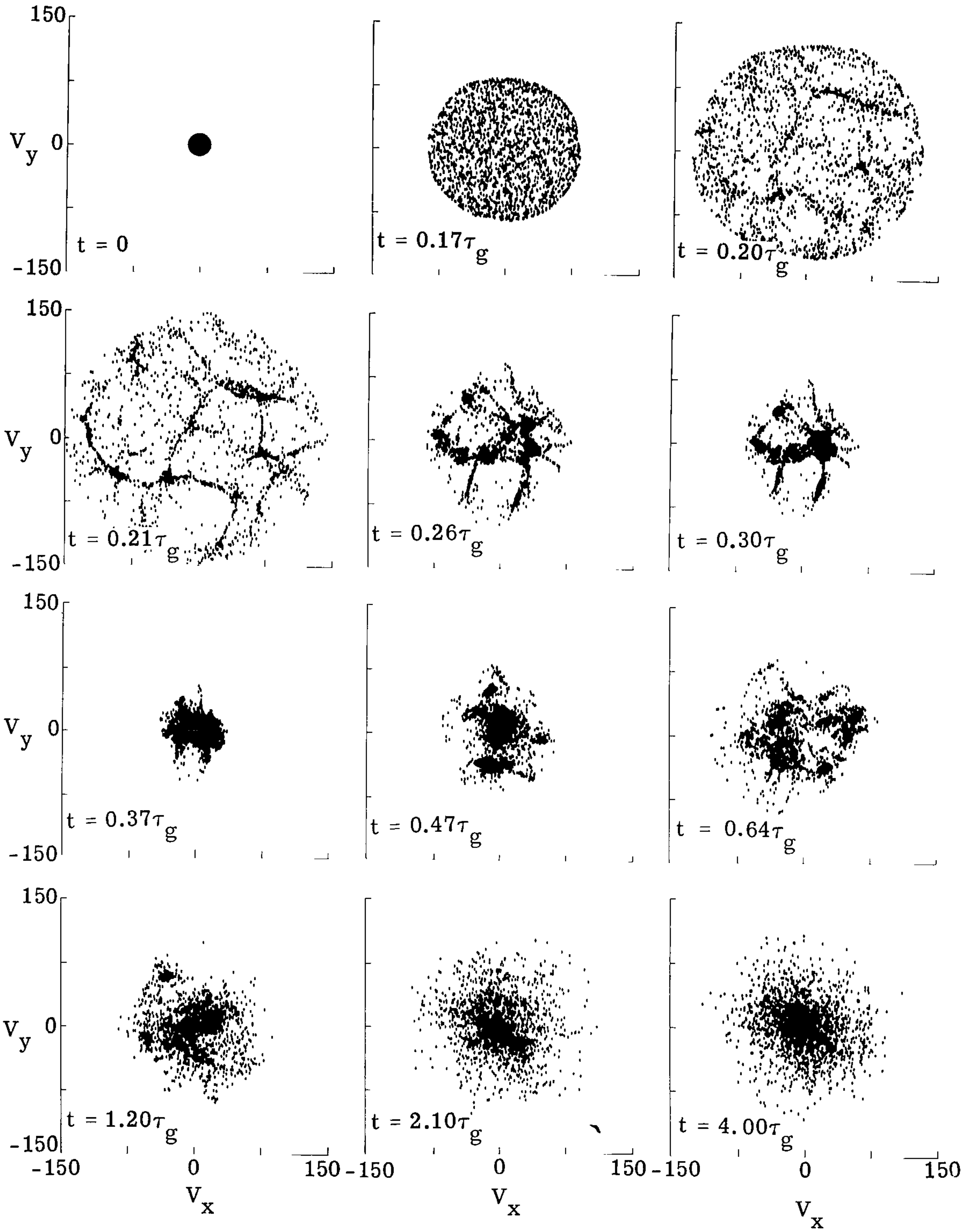
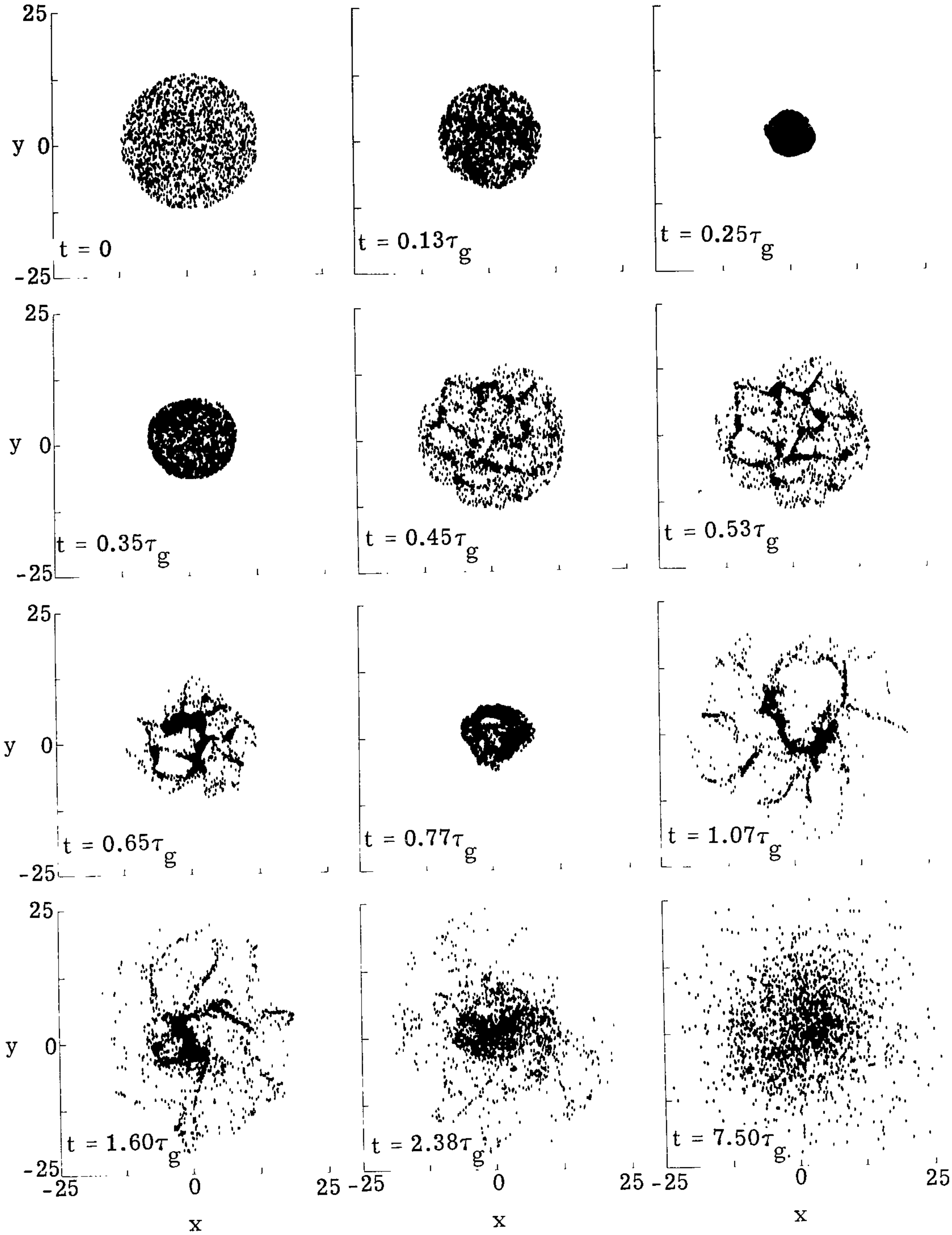
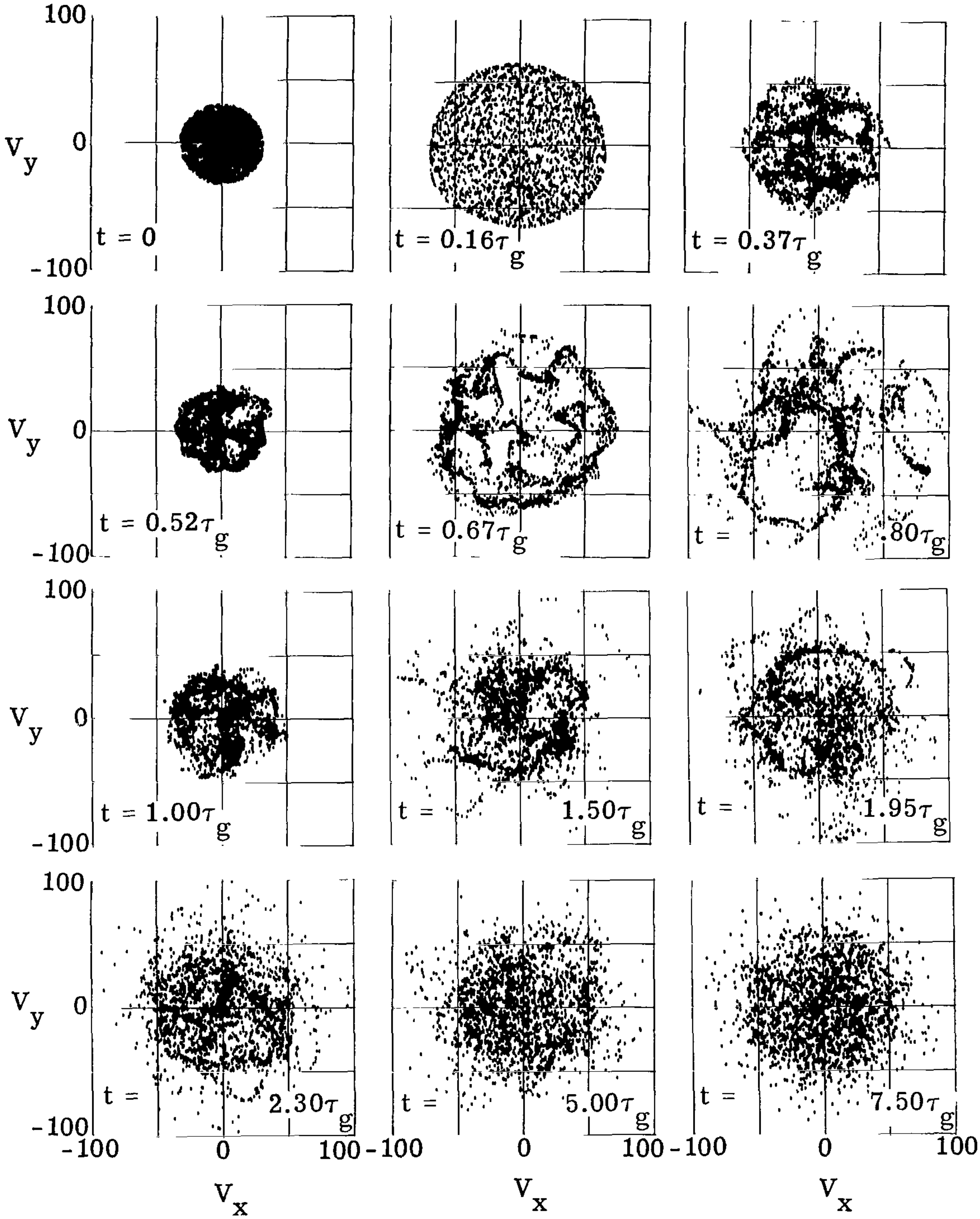


Figure 6.- Evolution in velocity space for a 2000-star system with $\omega = 0$.



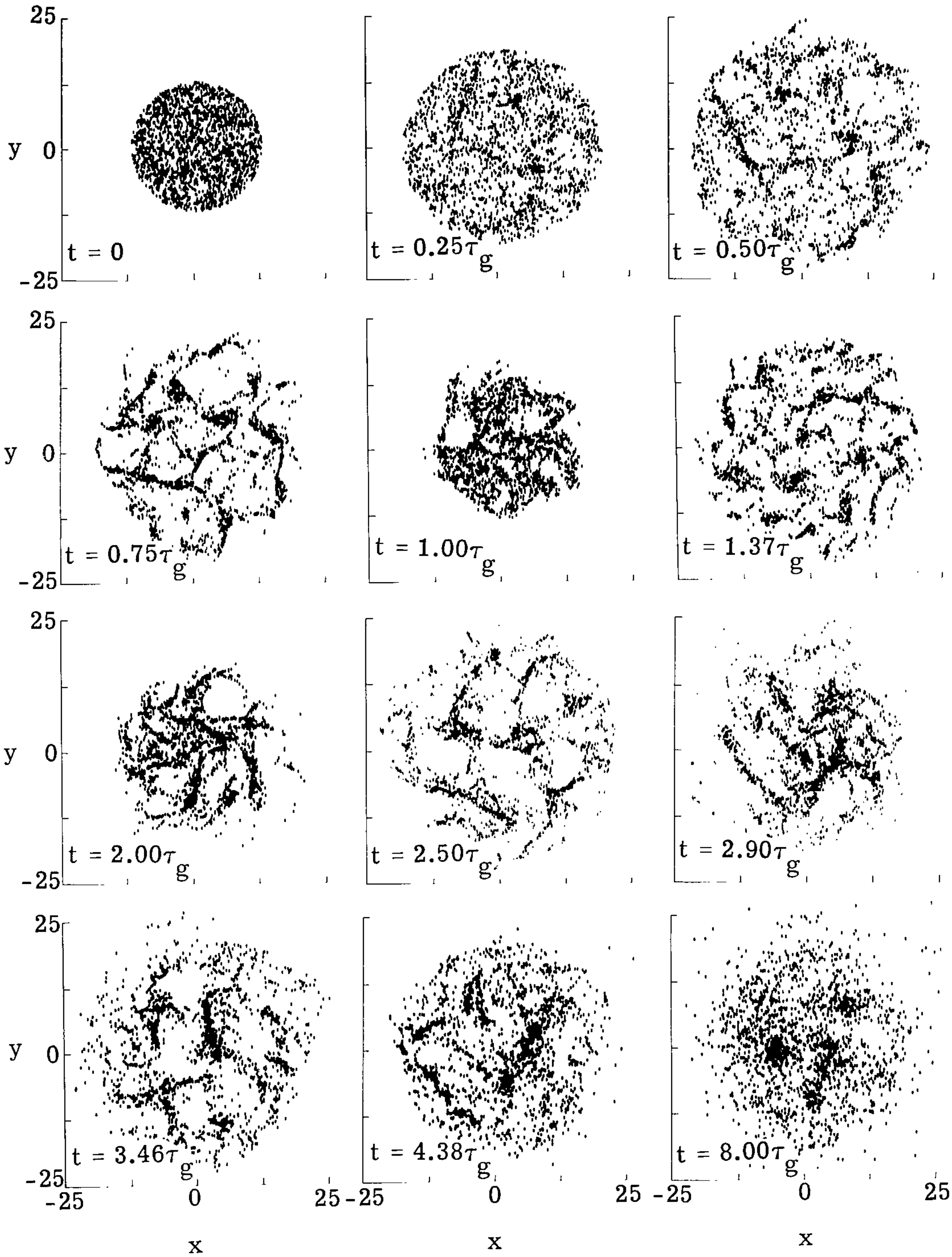
(a) In coordinate space.

Figure 7.- Evolution of a 2000-star system with $\omega = 0.5\omega_g$.



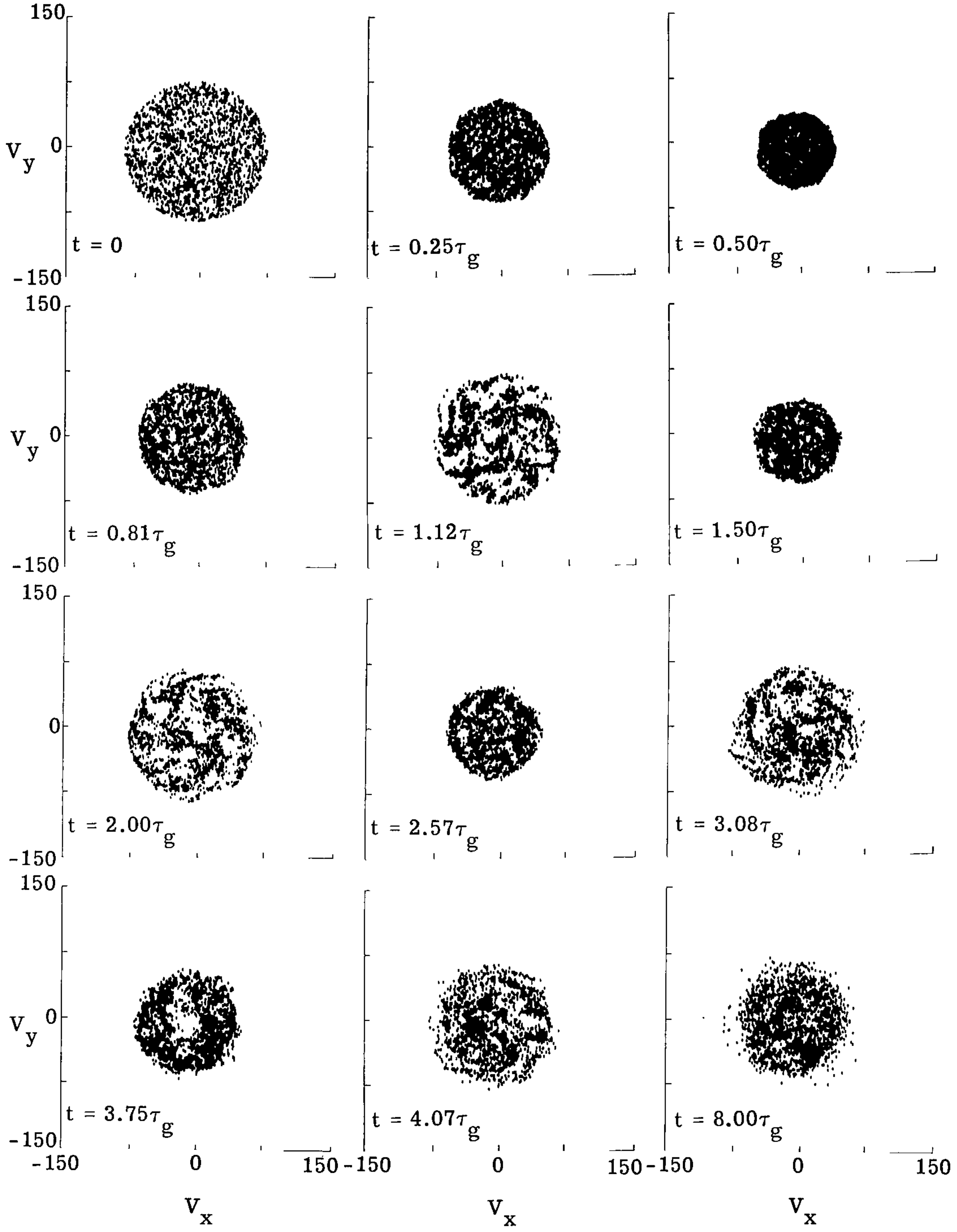
(b) In velocity space.

Figure 7.- Concluded.



(a) In coordinate space.

Figure 8.- Evolution of a 2000-star system with $\omega = 1.3\omega_g$.



(b) In velocity space.

Figure 8.- Concluded.

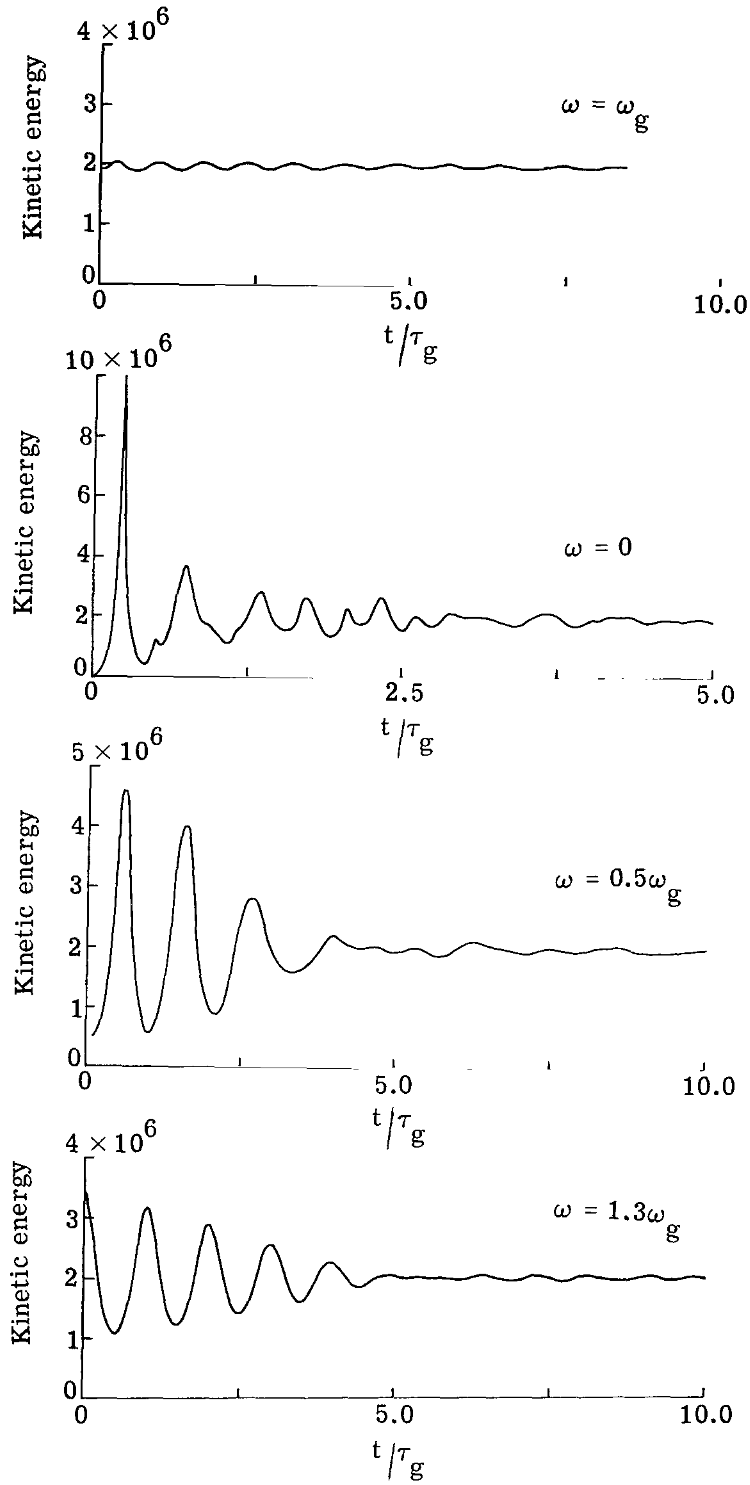


Figure 9.- Time development of the normalized total kinetic energy for 2000-star systems with various values of initial rotation.

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